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Annuities and Bond Discount*

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PART 2—BOND DISCOUNT

Under this section we shall discuss the question of bond discount and premium from the standpoint of the issuing company, as well as from the viewpoint of the investor in bonds purchased above or below the par value.

When a corporation decides upon an issue of bonds, the next question is: How are they to be disposed of? This is usually done by sale to or through some banking firm or syndicate of bankers, the manner of their disposal depending upon the amount of bonds issued, condition of the money market, and stability of the issuing company. If the bonds are sold directly to the bankers a definite price is generally received in payment, say 80, 90, 95, or more, as the case may be; the bankers then sell them to their customers at a higher price, thereby netting a profit on the transaction. Sometimes the issuing company pays a commission to brokers for disposing of the bond issue, or it may even sell them over its own counter. In any case there are certain heavy expenses incident to the issue of bonds, such as bond discount, commissions, advertising or other expenses, and these expenses must all be cared for on the books of the company. How shall it be done? The method to be adopted may depend upon the condition of the company's surplus account, though, of course, the directors in most cases have power to say what plan shall be used.

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If the company is financially strong, the entire expense incident to the bond issue may be charged to profit and loss at once, or it may be divided over a period of five or ten years; otherwise it is better to distribute it over the entire life of the bonds. It is common practice to amortize or spread such expense over the entire period for which the bonds are to run. Indeed, the special expenses incurred at the time of issue may all be charged off during the year incurred and only the bond discount be apportioned. For example, in the case of 5% bonds sold at 90 it is apparent that while the nominal rate of interest is 5% the effective rate is considerably higher; not only is the $5 interest to be paid annually for every $90 received, equaling a little over 5½%, but in addition to this the company must pay at maturity $10 more than it received. This amount should also be considered during the currency of the bond and part of it included each year as bond interest, thereby bringing the effective rate to something around 6%. The longer the bonds have to run, the lower will be the rate of bond discount to be amortized, while in an issue of short term bonds the yearly apportionment must necessarily be greater. The actual yield of such bonds to the investor and the true rate of interest paid thereon by the issuing company are difficult questions to decide, since the element of compound interest enters into such calculations; and, anyhow, this fineness of computation is unnecessary, since the universal custom is to charge bond discount to profit and loss periodically during the currency of the bonds, regardless of compound interest accumulations. In the case of a strong, well-managed corporation, it is a question whether the trouble occasioned by using compound interest formulas would be justified when the small difference in annual apportionments is considered.

The consideration of premium on bonds that have been sold above par is also of importance, since the principle is analogous to that of bond discount. Bond premium is not nearly so common as bond discount, but when it does appear it may also be distributed over the life of the bonds. Under ordinary conditions, however, it would seem prudent to dispose of it within the year when it occurred by a credit to either profit and loss or surplus, preferably the latter. It might even be entered as a credit to bond discount or some other deferred charge account.

The matter of bond discount has been pretty well defined by

the interstate commerce commission and by the various state public utility commissions, and in all cases its amortization during the life of the bonds has been sanctioned. Where both discount and premiums are to be considered, an account for discounts and premiums is permissible, so that only the balance need be distributed. In this connection the following quotation from Eaton's *Handbook of Railroad Expenses* (page 388) may be of interest:

UNEXTINGUISHED DISCOUNT ON SECURITIES

Ledger accounts should be raised to cover the discounts and premiums on each class of capital stock and each class of funded debt securities sold by the company. When securities which have been issued or assumed by the respondent company (except securities which have been sold and reacquired) are sold or exchanged by or for the respondent company for a consideration the actual money value of which, at the time of such sale or exchange, is less than the value of the securities at par and the accrued interest thereon, if any, the discount suffered should be debited to the account raised for discounts and premiums on securities of the class to which those sold belong. (Premiums realized from such sales or exchange should be credited to the account so raised.) If the net balances in the accounts covering discounts and premiums on the different classes of capital stock sold and the different classes of funded debt securities sold be debit balances, they should be included in the balance-sheet statement as follows:

(a) Unextinguished discount on capital stock—This balance should be carried on the balance sheet until extinguished by premiums realized on subsequent sales of stock by assessments levied on the stockholders, by appropriations of income or free surplus for the purpose r by retiring the stock. When any stock is retired the proper discounts and premiums account should be adjusted by crediting to it an amount equal to the unextinguished discount on such stock.

(b) Unextinguished discount on funded debt—At or before the close of each fiscal period, there should be charged to income (and credited to the discounts and premiums accounts in which the discount is carried) such proportions of the discount on outstanding debt obligations as may be applicable to that period. This proportion should be determined according to a rule the uniform application of which throughout the interval between the date of sale and the date of maturity will extinguish the discount on funded debt. In order that the discount may be extinguished earlier, the company may, at its option, charge to profit and loss all or any portion of the discount on funded debt remaining at any time unextinguished; but the charge to income for any period must not exceed the proportion applicable to that period and a charge should be made for each period so long as any portion of the discount remains unextinguished.

It will be of interest to note also what Mr. R. H. Montgomery says on these points:

DISCOUNT ON BONDS

When bonds are sold at a discount it is because the rate of interest the bonds bear is less than the effective rate at which the corporation's credit is rated. For instance, if 5 ten-year bonds are sold at 90, it means that the corporation's borrowing strength is rated at about 6%, and in order to reflect the actual rate each year as interest is paid, it would be necessary to carry the discount as a deferred charge among the assets and write off to interest account 1% annually. This, added to the amount paid in cash, will adjust the interest account to the proper cost.

PREMIUM ON BONDS

Where bonds are sold at a premium, the amount received in excess of the par value represents the equivalent of interest collected in advance, and must be held in reserve and distributed over the years to which it applies as a reduction in bond interest account. For instance, a corporation may sell its 5% ten-year bonds at 105, indicating that its credit is rated on a basis of about 4½%, that is, if a 4½ bond had been issued, the corporation should have realized about par. Therefore the bond interest, when paid, is subject to a deduction of ½ of 1% annually. The excess received at the time of sales should not be applied to income or to surplus, but, as stated above, must be carried as a deferred credit and reduced annually.

PLANS USED BY LARGE CORPORATIONS

By an inspection of recent annual reports issued by representative industrial and public service corporations, the policy of deferring such expenditures as bond discount and other extraordinary charges can be seen, and reference to a few of them is worth while.

The Public Service Corporation of New Jersey showed among its assets "unamortized debt, discount and expense," $4,262,504.10, after charging off $102,347.49 for the year. One of its subsidiaries, the Public Service Railway Company, shows among its assets unamortized discount and expense of $76,175.75, and among its liabilities unamortized premium on debt of $10,066.09.

The Studebaker Corporation shows among its assets "Discount and commission on 5% serial gold notes," $412,859.33, after writing off $93,773.02 for the year.

The Westinghouse Electric and Manufacturing Company in its last report carried an asset of $890,833.33 under "expenses incident to issue of bonds and notes," after having charged off $133,750.

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The Southern Pacific Company shows among its assets "discount on funded debt," $1,955,993.88, after writing off $505,476.92 during the year.

On the other hand, the Lehigh Valley Railroad Company in 1914 sold $10,000,000 of general consolidated mortgage 4½% gold bonds and charged all of the bond discount of $1,000,000 to profit and loss account, in reality against surplus.

Another indication of the desire to write off driftwood and to "fix-up" during the company's residence on "easy street" is seen in the Cambria Steel Company's report of 1913. The charges shown below were made against the surplus of the preceding year:

"After a careful study and examination of the properties and accounts as of January 1, 1913, it was deemed advisable and conservative accounting to set aside, out of the surplus accounts, aggregating $19,635,225.90, as shown December 31, 1912, the amount of $2,650,000 to cover the following adjustments:

First—Manufacturers Water Company: book value reduced....	$850,000
Second—By-product coke ovens: rebuilding fund not previously provided for—estimated cost......................	690,000
Third—Blast furnace relining account: to cover shortage due to certain current repairs charged to this account instead of to costs.....................................	110,000
Fourth—Converting works, open hearth steel plants, blowing engines, mills, etc.: replacements—approximate cost.	1,000,000
Total	$2,650,000

Capitalizing Bond Discount

It is contended by many corporation officials that bond discount and all expenses incident to the bond issue should be capitalized—that is, charged to the plant or buildings for which the money derived from such bond issue is to be used. It is claimed by them that the cost of constructing such a plant must obviously include not only the cash proceeds but all expenses incurred in securing the necessary capital; indeed, this principle is strongly advocated by the treasurer of one of the well-known companies mentioned above.

This plan may have some merit, since the charge thus capitalized will be written off in the form of depreciation in the same manner as it would have been handled had it been placed in a separate account and distributed over the life of the bonds. It is a question, however, whether expenses incurred in financing

the business through bond issues should be included in the cost of production, as it manifestly would be were it charged to plant and then written off in the form of depreciation. We have already seen that bond discount is usually coupled with bonds which bear a low rate of interest and that such bond discount by being spread over the currency of the bonds is considered part of the interest. Therefore, since interest on bonds is not considered a production cost, neither should bond discount be regarded as such. Mr. A. Lowes Dickinson has the following to say on this subject:

Discount on Bond Issues Not a Proper Charge to Capital

Under no ordinary circumstances is it correct to treat discount on bond issues as a charge to capital. It does not represent any property; for it is clearly incorrect to consider that the cost in cash of a piece of property varies according to the credit of the purchaser or constructor. Hence, where any such discounts have been charged to capital assets, it is essential for a proper understanding of the balance sheet that this fact should be clearly set forth. In effect, in corporation finance such discounts are frequently included in capital expenditures as are many other fictitious items which require either a sanguine estimate of future gains, or mere "water"; and this is done by the fiction of the sale of the property to a new company at a largely inflated value, for a price including the par value of the bonded debt; the intermeidaries in this sale then selling the bonds at the market price. This principle has been extended also to cases of reorganization effected without any sale of assets, where the discounts on bonds, as well as bonuses of capital stock issued to facilitate the completion of the reorganization, have all been capitalized. These exceptions must not be taken as a justification for a similar treatment in ordinary cases of financing; and, in fact, there is no real justification for their adoption in any case, beyond that of expediency. The whole system has led in the past to many and serious abuses, and the more strict supervision of capital issues which is now being extended over certain classes of corporations will tend to put a stop to it in all cases. The objection to charging bond discount to capital account must not, however, be held to include such a charge of the annual instalment of discount for the period of construction during which interest also is charged to capital account. The rate of interest chargeable to capital should be the actual cost of the money, i.e., the effective rate, and any approximation to this rate obtained by including that proportion of the discount which belongs to the period would be a permissible charge to capital.

Construction Costs

In this connection it may be of interest to note that the interstate commerce commission permits the capitalizing of expenses incurred during the construction period, especially of those expenses which appertain to and are part of the construction cost.

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These expenses include interest on bonds and other obligations, law charges, injuries to persons, miscellaneous construction expenditures, and such other items of expense as may occur in connection with the work. The commission permits also the addition of bond discount to the construction cost, but obviously only such proportion thereof as the period of construction bears to the life of bonds. While industrial and other companies are not subject to commission regulations, they are at liberty to follow whatever plan seems most desirable. If the plan is a good one for transportation companies it might be considered equally serviceable for industrials, especially if conditions were alike.

Turning to the requirements of state utility commissions, we find some variations. All the commissions agree that certain expenses incurred during the construction period may be included as part of the construction cost, but most of them prohibit the addition of bond discount. The public service commission of New York does not permit the inclusion of bond discount in the construction cost, but sanctions its amortization over the life of the bonds.

At the date of issue the bond discount and the expenses incident thereto are required to be charged to "unamortized debt discount and expense." At the end of each fiscal year (or month) the required portion to be written off is credited back to this account and debited to "amortization of debt discount and expense," which account in turn is closed into profit and loss as a deduction from income. Bond premium, on the other hand, is to be set up as a credit to an account called "unamortized premium on debt" and then written off in yearly (or monthly) proportions to "amortization of premium on debt." The schedules of classifications and annual report blanks of the New York commissions can be obtained by payment of a nominal fee, but in many of the states they are available for free distribution.

Premium and Discount on Investments

Investors in bonds frequently purchase them either above or below the par value, depending, of course, upon the time they have yet to run and the rate of interest which they will draw. Bonds purchased above par manifestly return a lower rate of interest than the coupons call for, and conversely, bonds purchased below par return a higher rate. In determining the true

return the rate received by an investor necessitates the amortization of such premium or discount over the life of the bond and either added to or deducted from the cash income at each interest period. In order to illustrate the principles involved in determining the effective rate in amortizing the discount or premium, a few illustrations are given below:

Example 13—A school district issues bonds for $12,000, bearing interest 6% payable yearly and to run 5 years. For what sum should they sell, money being worth 5% yearly?

Answer—In this case there are really two questions to solve. The debenture at face value of $12,000 is to be paid at the end of 5 years. The interest, 6% of $12,000, gives us a figure of $720 to be paid each year for 5 years. The two results added give us the complete present worth:

$$12,000 \times \frac{1}{(1.05)^5} \,\&\, \frac{720}{.05} \left(1 - \frac{1}{(1.05)^5} \right) = \$12,519.54$$

They should sell for $12,519.54.

Example 14—What should be paid for a $1,000 coupon bond maturing five years hence and bearing 4% interest payable annually, so that the investor will receive 5% per annum compound interest on his money?

Answer—$956.71. The solution to this problem may be stated on the principle followed in the preceding example.

Amortization and Its Application

The example which follows is used to illustrate the method of handling premium and discount on bonds, and the manner of determining the rate of yield on bonds purchased at a given price.

Amortization means the general reduction or extinguishment of a debt or of a given sum. The sinking fund or annual payments for the redemption of a bond or mortgage may be cited as an example.

A common instance of amortization, however, is applied to a bond which has been purchased at a premium and redeemable at par after a term of years. While the bond draws a certain rate of interest, this could not be considered entirely as a profit, inas-

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much as the premium must be written off over a term of years and part of said premium charged against the annual or semi-annual income. The total income or interest received, less the amount deducted annually for amortization, is the net income. The annual income is termed the nominal interest; the balance after provision has been made for reduction of premium is the effective interest or true yield of the investment.

Suppose $105 to have been paid for a $100 bond, redeemable in ten years, bearing interest at 5 per cent per annum. The $5 received each year is the nominal interest and estimated on the par value of the bond, but the true or effective interest yield is somewhat lower and determined only after the correct portion of premium has been deducted.

The term "amortization" has come into wide use during the past few years and is now being used systematically where formerly it was disregarded; yet the method of handling it is complicated and is of little use unless one is thoroughly acquainted with its operation and the manner of computing same. Because of this inability, tables of bond values have been prepared which give the information desired in connection with bonds purchased at different quotations and bearing different rates of interest. These tables can be purchased and should be in the hands of any person who has much to do with this class of securities.

Determining the Rate of Yield

To make entries for amortization values in books of account it is necessary to know the rate of yield on the bond under consideration. Therefore, the first thing to determine is the true rate of interest yielded after the deduction of premium. This may be had from the bond tables or estimated by the use of the annuity and compound interest tables. By inspection one may determine very closely the rate of yield that the bond will give, but to prove the accuracy of this assumed rate it is necessary by successive trials to (1) find the present worth of the face of the bond and (2) the present worth of the annual or semi-annual interest instalments. The amount of these two results, if computed at the correct rate of yield, will equal the cost of the bonds.

To determine the purchase price of a bond yielding a given nominal rate in order to realize a given return on the outlay, the same method is followed as shown in the preceding illustration,

example 13, in which the necessity of successive trials does not obtain.

If the bond is purchased at a discount instead of at a premium, the same principle is used as in the case of premium, but the true yield in such a case would be higher than the nominal rate of interest.

The Book Entries

After determining the rate of yield on the bond the next step is to get the amount to be charged off each year. The entries thereafter are not unusual. When the bond is purchased, debit bond account for the cost thereof. This will include the premium, of course. At the end of each year the bond account should be credited with the amount of amortization. The balance shows the new amount to be carried forward to the following year, which in turn will be credited at the end thereof with the amount to be written off. The process is continued from year to year, and when the bond matures it will have been reduced by amortization to the par value which is then paid in cash. Another method is to charge a premium account with the amount paid in excess of the par value of the bond. From year to year this account should be credited with the amount of amortization and the balance carried forward to the next year. By this method the premium is gradually written off over the term of years which constitutes the currency of the bond.

If the bond is purchased at a discount, debit the bond account for the cost as explained above. At the end of each year the bond account should be debited with the amount of amortization and thereby gradually increased to the par value thereof. In that case the entries for amortization shown in the illustration would be reversed. It is unwise, however, to write up too freely unless there is a certainty that the bond will be paid in full at maturity.

Example 15—You purchased January 1, 1915, a $10,000 6 per cent bond, having three years to run, for $10,275. If the coupons are payable semi-annually, what per cent did you make on your investment? Show the entries involved, and the records for bond and amortization for the entire time.

Answer—An answer sufficient for ordinary purposes may be obtained by dividing the bond premium by the number of years

the bonds have to run, or better still, to agree with income dates by the number of half-years they have to run. The half-yearly income is $300, and one-sixth of the premium is $45.38; this deducted leaves a net income of $254.17, or $509.34 per annum. On this basis the net annual income would seem to be considerably over 5 per cent. This is not just true, as will be seen below, but it serves as a guide in estimating the rate of yield and in many cases might be considered near enough. Compound interest was not considered, nor interest on the premium, of the use of which the investor has been deprived. Now consider the matter from a scientific standpoint. The experimental method used below in determining the true rate of yield may be criticized by mathematicians, but it is no doubt more useful for ordinary purposes than difficult algebraic formulas. The answer given to the question agrees with the bond tables. By consulting the bond tables, the yield on any kind of bond bought at any given price can be readily determined. That is why the tables are so useful to bankers and investors.

In answering the question, we first determine what rate of yield will be realized on a 6 per cent bond due in three years, bought for a given amount. The rate of interest desired is that rate at which the present worth of the interest and of the principal added together will amount to $102.75 on each bond. To arrive at the correct rate necessitates certain experiments. Let us try 5½% and make use of the annuity and compound interest tables:

Present worth of $1 annuity for 6 years at
   2¾%   = $5.46236678

Present worth of $1 compounded for 6 years
   at 2¾%   =   .84978491

Then on this basis the

Present worth of $300 income annuity = 300
   × 5.46236678   = $ 1,638.71

And the present worth of $10,000 compounded = 10,000 × .84978491   =   8,497.85

Present worth of both bond and interest = $10,136.56

We can see by the above that the annual rate of interest is too high, since it reduces the present worth of the bond and in-

terest instalment to $10,136.56, while the actual present worth or cost is $10,275.00. Then we must try a lower rate of interest and keep on trying until a rate is obtained that will give a present worth of $10,275, or near enough to it for practical purposes.

Let us try 5% this time, or 2½% on half-yearly intervals:

*Present worth of $1 annuity for 6 years at
  2½%   = $5.508125
*Present worth of $1 compounded for 6 years at
  2½%   =   .862296
Then the
  Present worth of $300 annuity =
    5.508125 × 300   == $ 1,652.44
  and the present worth of $10,000 com-
    pounded = .862296 × 10,000 =   8,622.96
                                    _____

Present worth of bond and interest = $10,275.40

We see that the above is correct and that the result is within 40 cents of the cost of the bond; the 40 cents may be spread over the life of the bond or else adjusted either in the opening or closing year.

The rate of yield, as proven above, is 5 per cent on the cost of the bond, though the cash income each half year is $300. The true income is 2½% of $10,275, or $256.88. The $300 less $256.88 gives $43.12, the amount of amortization or sum to be charged off against premium at the first interest period. This amount deducted from $10,275 leaves $10,231.88, on which the next periodic income is based. Two and one-half per cent of $10,231.88 gives $255.80. Three hundred dollars less $255.80 gives $44.20, the amount of amortization for the second period. The same operation is performed each period during the life of the bond, and the amount of amortization charged against the bond premium. Instead of charging the entire investment of $10,275 to investment account, as shown below, $10,000 might be charged to investment and $275 to premium account. In that case the premium account would be written down as illustrated, while the investment account would remain untouched until maturity.

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*Amounts taken from the annuity tables, but may be determined from compound interest tables.

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The book entries in journal form and without providing for monthly accruals are as follows:

January 1, 1915

Bond investment...........$10,275.00		
To cash................		$10,275.00

For purchase of $10,000, 6 per per cent bond due in three years.

July 1, 1915

Cash	300.00	
To income..............		300.00

For interest coupon on bond investment.

Income	43.12	
To bond investment.......		43.12

For portion of bond premium written off by amortization.

January 1, 1916

Cash	300.00	
To income..............		300.00

For interest coupons.

Income	44.20	
To bond investment.......		44.20

Amount of amortization charged to premium.

July 1, 1916

Cash	300.00	
To income..............		300.00

Income	45.31	
To bond investment.......		45.31

January 1, 1917

Cash	300.00	
To income..............		300.00
Income	46.44	
To bond investment.......		46.44

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July 1, 1917

Cash	300.00	
To income..............		300.00
Income	47.60	
To bond investment.......		47.60

January 1, 1918

Cash	300.00	
To income..............		300.00
Income	48.33	
To bond investment.......		48.33
Cash	10,000.00	
To bond investment.......		10,000.00

For payment of 6 per cent bond maturing this date.

The Ledger Accounts

Bond Investment Account

(Alien Gas Company 6% bond due January 1, 1918, to net 5% interest)

1915		1915	
Jan. 1 To Cash.....$10,275.00		July 1 By income...$	43.12
		1916	
		*Jan. 1 By income..	44.20
		July 1 By income...	45.31
		1917	
		Jan. 1 By income...	46.44
		July 1 By income...	47.60
		1918	
		Jan. 1 By income...	48.33
		Jan. 1 By cash.....	10,000.00
$10,275.00			$10,275.00

Note—If desired, the account may be balanced at the end of each period and the balance brought down. * By right income adjustment should be made on December 31st.

Income Account

1915		1915	
July 1 To bond investment	$ 43.12	July 1 By cash........	$300.00
July 1 To profit and loss	256.88		
	$300.00		$300.00

Note—The same operation should be performed each half year, or if desired the account may remain open until the end of the year, and then be closed into profit and loss.

In case the premium is kept in a separate account, and only the par value entered in the bond account, the two accounts would appear as follows:

Bond Investment
(6% bond cost $10,275, to net 5%)

1915		1918	
Jan. 1 To cash.....	$10,000.00	Jan. 1 By cash.....	$10,000.00

Bond Premium
(On $10,000 6% bond investment of 1918)

1915		1915	
Jan. 1 To cash........	$275.00	July 1 By income....	$ 43.12
		Dec. 31 By income....	44.20
		1916	
		July 1 By income....	45.31
		Dec. 31 By income....	46.44
		1917	
		July 1 By income....	47.60
		Dec. 31 By income....	48.33
	$275.00		$275.00

Bonds purchased below par

Example 16—To illustrate the manner of amortizing the discount on bonds purchased below par, we will assume that

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a \$10,000 bond having five years to run was purchased on January 1, 1915, for \$9,573.25. Assuming that the company is thoroughly trustworthy and that the bond will be paid in full at maturity, the accounts for investment and income would be about as follows:

Bond Investment Account
(Central Electric Company 5% of 1920, yielding 6%)

1915		1920	
Jan. 1 To cash.....$	9,573.25	Jan. 1 By cash.....$10,000.00	
July 1 Amortization.	37.20		
1916			
Jan. 1 Amortization.	38.31		
July 1 Amortization.	39.46		
1917			
Jan. 1 Amortization.	40.65		
July 1 Amortization.	41.87		
1918			
Jan. 1 Amortization.	43.12		
July 1 Amortization.	44.42		
1919			
Jan. 1 Amortization.	45.75		
July 1 Amortization.	47.12		
1920			
Jan. 1 Amortization.	48.85		
	$10,000.00		$10,000.00

Income from Bond Investment

1915		1915	
July 1 To profit and loss.$287.20		July 1 By cash........$250.00	
		Amortization....	37.20
1916		1916	
Jan. 1 To profit and loss. 288.31		Jan. 1 By cash........	250.00
		Amortization....	38.31
July 1 To profit and loss. 289.46		July 1 By cash........	250.00
		Amortization....	39.46

1917		
Jan. 1 To profit and loss.	290.65	

1917		
Jan. 1 By cash........	250.00	
Amortization....	40.65	

July 1 To profit and loss.	291.87

July 1 By cash........	250.00
Amortization....	41.87

1918	
Jan. 1 To profit and loss.	293.12

1918	
Jan. 1 By cash........	250.00
Amortization....	43.12

July 1 To profit and loss.	294.42

July 1 By cash........	250.00
Amortization....	44.42

1919	
Jan. 1 To profit and loss.	295.75

1919	
Jan. 1 By cash........	250.00
Amortization....	45.75

July 1 To profit and loss.	297.12

July 1 By cash........	250.00
Amortization....	47.12

1920	
Jan. 1 To profit and loss.	298.85

1920	
Jan. 1 By cash........	250.00
Amortization....	48.85

Note—It is more than likely that many of the above entries would bear dates of June 30th and December 31st, instead of July 1st and January 1st.

Another Method of Solving

Here is another method of working out Examples 15 and 16 that has been presented by one of my students in accountancy, Mr. D. J. McRae. It is of interest because of the way in which the correct rate of interest is determined. The method is applied to both of the problems, as follows:

Second Solution of Example 15

"There is a premium of $275 (2¾% of the par value) to be spread over a period of six half years at 3% (6% per annum). The problem, therefore, is to find what annuity will amount to, or have a final value of, $275.00 in six years at 3%.

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275 = final value of unknown quantity

6.46841 = final value of $1.00 annuity in 6 years at 3%

$\dfrac{275}{6.46841} = 42.51$, unknown quantity.

$42.51 is therefore the amount to be deducted each period from the income from the bonds. For so short a period I call this $42.50, as that will be near enough for practical purposes. Not allowing for interest accumulating, this will amount in six periods to $255.00. During these six periods the nominal income from the bonds will amount to $1,800 ($300 each six months). The $1,800 minus $255 = $1,545.00 = net income for 3 years. $1,545 ÷ 3 = $515 = net income for 1 year ($1,545 is approximately correct).

$\dfrac{\$515}{10.275} = .05 + = 5\%$, the effective rate of income.

"Five per cent is not exact, but it is so close as to bring a discrepancy of only 46 cents at the end of three years. I then use the following scientific amortization of the premium, using 5% as the effective rate:

Proof of Effective Rate (5%)

	Interest.				Bonds.	
Peri-ods ½ year	Nominal 6% on par.	Effective 5% on cost.	Amortiza-tion of cost to par.	Dr.	Cr.	Balance.
1	$300.00	$256.88	$43.12	$10,275.00	$43.12	$10,231.88
2	300.00	255.80	44.20		44.20	10,187.68
3	300.00	254.69	45.31		45.31	10,142.37
4	300.00	253.56	46.44		46.44	10,095.93
5	300.00	252.40	47.60		47.60	10,048.93
6	300.00	251.67	48.33		48.33	10,000.00
	$1,800.00	$1,525.00	$275.00	$10,275.00	$10,275.00	

"In the above, the effective interest is the interest on the last balance (or inventory value) of the bond, taken at the effective rate. The amortization instalment is the difference between the nominal and true or effective interest. Each successive instalment is deducted from the preceding balance, and a new principal thus obtained."

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Second Solution, Example 16
Finding the Rate

Period ½ year	Amortization of discount.	Interest payments.	Deferred balance of bond discount.	Semi-annual charges to profit & loss
1	$42.68	$250.00	$426.75	$10.67
2	42.67	250.00	348.07	9.60
3	42.68	250.00	341.40	8.54
4	42.67	250.00	298.72	7.47
5	42.68	250.00	256.05	6.40
6	42.67	250.00	213.37	5.33
7	42.68	250.00	170.70	4.27
8	42.67	250.00	128.02	3.20
9	42.68	250.00	85.35	2.13
10	42.67	250.00	42.67	1.07
Total ..	$426.75	$2,500.00		$58.68

$2,500 = income from interest.
426.75   income from discount.
$2,926.75 total income.
58.68   loss from deferred discount.

$2,868.07 net income for five years.

$2,868.07 ÷ 5 = 573.61, net income for 1 year.
$573.61 ÷ 9,573.25 = .0598 = 5.98% effective rate.
For practical purposes call this 6%.

Then we have below:

Periods ½ year.	Interest.			Bonds.		
	Nominal 5% on par.	Effective 6% on cost.	Amortization to par.	Dr.	Cr.	Balance.
				$9,573.25		
1..........	$250.00	$287.20	$37.20	37.20		$9,610.45
2..........	250.00	288.31	38.31	38.31		9,648.75
3..........	250.00	289.46	39.46	39.46		9,688.23
4..........	250.00	290.65	40.65	40.65		9,728.87
5..........	250.00	291.87	41.87	41.87		9,770.74
6..........	250.00	293.12	43.12	43.12		9,813.86
7..........	250.00	294.42	44.42	44.42		9,858.28
8..........	250.00	295.85	45.75	45.75		9,904.03
9..........	250.00	297.12	47.12	47.12		9,951.15
10..........	250.00	298.85	48.85	48.85	10,000.00	
Total.....$2,500.00		$2,926.75	$426.75	$10,000.00	$10,000.00	

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Objection to the Estimating Plan

There is considerable objection to the experimental method of determining the rate of return on the investment; especially with the idea of making repeated trials in arriving at the effective rate of interest earned. It does seem like unnecessary work, but the plan is one that can be followed by any person with a knowledge of annuities and compound interest; indeed, even with the aid of algebra some method of approximating has to be resorted to in obtaining the effective rate. There is an algebraic formula, I believe, that will produce a correct answer, but it is too difficult for ordinary usage. In the purchase of a bond for a given figure we gain the right to receive the face value at maturity as well as definite amounts (annuities) every interest period. If the security were purchased at par the rate of return would be that shown in the face, but when bought at a premium or discount the effective rate must obviously change. This rate can, of course, be readily determined from a table of bond values, but that does not enlighten one on the manner of arriving at the correct solution.

In writing on this point, Mr. Sprague, in his *Accountancy of Investment,* stated that "in probably the majority of cases the bargain is made 'at a price' and income rate must be found. This is a more difficult problem than where the bond has been bought on the basis of given returns. To ascertain the basis, when the price is given, is by trial and approximation—in fact, all methods more or less depend upon that. The ordinary tables will locate several figures of the rate, and one more figure can safely be added in settling proportions. But it is an important question to what degree of fineness we should try to attain. Where the rate per cent of return involves an unwieldy fraction or decimal, it is customary to use an even per cent and then to apportion the balance over the life of the investment in some equitable manner."